Seminar on
Statistical Extreme Value Theory

winter term 2024/25

Prof. Dr. Axel Munk

Key information

Time:	01/11/2024 – 07/02/2025, on Fridays, 10.15 – 11.45
Format:	Seminar room 5.101 (Institut für Mathematische Stochastik)
Possible Modules:	B.Mat.3444: Seminar on mathematical statistics
	M.Mat.4844: Seminar on mathematical statistics
	B.Mat.3447: Seminar on statistical foundations of data science
	M.Mat.4847: Seminar on statistical foundations of data science
Instructors:	Michel Groppe and Prof. Dr. Axel Munk
Intended Audience:	**Advanced Bachelor and beginning Master students**
Language:	English

Prerequisites

Participants must have successfully attended:

- Measure & probability theory (B.Mat.1400).

Description

(https://commons.wikimedia.org/wiki/File:Large_breaking_wave.jpg)

For many tasks of modern data analysis the stochastic modeling of very rare events, such as observing extremely low/high values or a value larger/smaller than any of the previously observed values, is important in many applications. For example, consider modeling and predicting the height of tidal waves in the planning phase of building a dam, determining future claim heights when developing policies how much money insurance companies have to set aside, prediction of records in sports, or modeling the oldest age or largest body height of people from a specific population.

For all of these questions the statistical behavior, in particular the distribution, of the maximum $M_n = \max\{X_1, \ldots, X_n\}$ of a sample $X_1, \ldots, X_n$ is to be considered.

In contrast, Central Limit Theorems (CLTs) allow us to approximate the distribution of "average" type statistics that are (related to) the sum $S_n = X_1 + \ldots + X_n$ obtained from a random sample. More precisely, under assumptions, we then have that $P((S_n - \mu_n)/\sigma_n \leq x) \to \Phi(x)$, as $n \to \infty$, where Φ is the c.d.f. of a standard normal variable, $\mu_n = \mathrm{E}(S_n)$ and $\sigma_n^2 = \mathrm{Var}(S_n)$. One of the aspects that makes CLTs so useful is that the limit is always $\Phi(x)$, independent of the specific distribution of the X_i's.

However, the behavior of extremes is substantially different. Extreme Value Theory provides asymptotic results of the form $P((M_n - a_n)/b_n \leq x) \to G(x)$ and it turns out that G is always of the form

$$G(x) = \exp\Big(- \Big[1 + \xi\Big(\frac{x-\mu}{\sigma}\Big)\Big]\Big),$$

a so-called generalized extreme value distribution with location, scale and shape parameters μ, σ, ξ.

In this seminar we cover the basic relevant mathematical concepts and main results of extreme value theory. Our focus will be on carefully understanding the basic principles rather than obtaining results in most generality. Topics include: extreme value distributions, Cauchy's functional equation, Fisher-Tippett-Gnedenko theorem, regular variation, max-domain of attraction, and statistics for extreme value distributions. Finally, we discuss some applications and statistical estimation techniques for generalized extreme value distributions.

This seminar complements the lecture Foundations of Statistical Data Science I (B.Mat.3147) by Prof. A. Munk.

Application

To provide participants with the material to be presented at an early stage, we ask you to preregister for this seminar. To this end, please email Anja Rentzsch (anja.rentzsch@uni-goettingen.de) and indicate your interest to give a seminar talk. Please include information about relevant courses you have taken in your email. Deadline for preregistration is 30 August 2024 (12pm/noon).

A preparatory virtual meeting, during which topics will be assigned to participating students, is scheduled for 2 September 2024 (2pm–3pm). Notably, the seminar is limited to 14 participants. Should preregistrations exceed 14, then participants will be chosen based on the information provided in their preregistration email.

Recommended literature

Main Reference

Topics for presentations will be assigned along the lines of

- Z. Kabluchko: Lecture Notes "Extremwerttheorie". Available for download via
  `https://www.uni-muenster.de/Stochastik/kabluchko/Skripte/Skript_Extremwerttheorie.pdf`

References for further reading

- P. Embrechts, C. Klüppelberg, T. Mikosch. Modelling Extremal Events for Insurance and Finance.
- S. Resnick. Extreme Values, Regular Variation and Point Processes.
- L. de Haan, A. Ferreira. Extreme Value Theory: An Introduction.
- S. Resnick. Heavy-Tail Phenomena: Probabilistic and Statistical Modeling.
- M. Falk, J. Hüsler, R.-D. Reiss. Laws of Small Numbers: Extremes and Rare Events.
- J. Beirlant, Y. Goegebeur, J. Segers, J. Teugels. Statistics of Extremes: Theory and Applications.